

The Evolution of a Hard Graph Theory Problem – Secure Sets

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A Secure Set of a graph is a set S of vertices with the property that for every subset X of S , $N[X] \cap S$ contains as many vertices as there are neighbors of X not in S . This is a special kind of Defensive Alliance, which only requires the property to hold for all singleton subsets of S . The question of whether a graph G has a Defensive Alliance of no more than k vertices, for arbitrary G and k , is known to be NP-Complete. The similar question for Secure Sets seems to be much harder than any NP-Complete problem. In fact, if $P \neq NP$, it is probably not even in the set NP. On the other hand, if $P = NP$, it can be solved by polynomial algorithms.

How this all comes about and at least part of what it means will be the topic of this presentation.