

**2004-2005**

**MATHEMATICS COLLOQUIUM SERIES**

03/01/05 Colloquium

**Prof. Zixia Song  
Ohio State University**

will speak on:

**Extremal Functions for Contractions of Graphs**

A graph  $H$  is a minor of a graph  $G$  if  $H$  can be obtained from a subgraph of  $G$  by contracting edges. One of the central problems in Graph Theory is Hadwiger's conjecture, which states that for every integer  $t > 0$ , every loopless graph with no  $K_t$  minor is  $(t-1)$ -colorable ( $K_t$  denotes the complete graph on  $t$  vertices). For  $t > 4$ , Hadwiger's conjecture implies the Four-Color Theorem. The cases  $t=5,6$  are, in fact, equivalent to the Four-Color Theorem, but Hadwiger's conjecture is open for all  $t > 6$ . The above equivalence for  $t=6$  makes use of the following beautiful theorem of Mader: for every integer  $p=1,2,\dots,7$ , a simple graph on  $n$  vertices and at least  $(p-2)n - (p-1)(p-2)/2 + 1$  edges has a  $K_p$  minor. Mader's theorem does not extend to  $p > 7$ , but Jorgensen characterized all counterexamples for  $p=8$ . Seymour and Thomas conjecture that Mader's theorem holds for all  $p$  and sufficiently large  $(p-2)$ -connected graphs. Jointly with Robin Thomas, we prove that Seymour and Thomas' conjecture is true for  $p=9$  by explicitly describing all counterexamples to Mader's formula. We have also studied counterexamples to Mader's formula for  $p=10$  and believe that our methods can be used to extend Mader's theorem for the case when  $p=10$ . In a related work I have established a conjecture of Jakobsen from 1983 concerning the extremal function for the graph  $K_8$  with one edge deleted. I will also mention a recent joint result with K. Kawarabayashi.

Dr. Song received the Ph.D. in Mathematics from Georgia Institute of Technology in June, 2004. Presently, she is a Zassenhaus Assistant Professor at Ohio State University. Her primary research interest is graph theory.

**DATE: Tuesday, March 1, 2005**  
**TIME: 10:30 – 11:30 AM**  
**PLACE: Math and Physics Building, Room 233**