Asymptotic Study of Orthogonal Laurent Polynomials and Related Quantities

Abstract

Asymptotic formulae for orthogonal polynomials of various types used in numerical calculations in the limit of large numbers of discretization points are crucial in obtaining sharp error estimates. In this presentation, asymptotics of the orthogonal Laurent polynomials (OLPs) $\phi_{2n}(z) = \sum_{k=-n}^{n} \xi_k^{(2n)} z^k$ and $\phi_{2n+1}(z) = \sum_{k=-n-1}^{n} \xi_k^{(2n+1)} z^k$ are derived as $n \to \infty$ for any complex z. The OLPs follow from the Gram-Schmidt orthogonalization of $\{1, z^{-1}, z, z^{-2}, z^2, \dots, z^{-k}, z^k, \dots\}$ when the inner product involves an exponential weight. Asymptotics are also obtained for the coefficients of recurrence relations that these OLPs satisfy, as well as for associated Hankel determinant ratios.

The asymptotics are obtained by formulating each OLP in terms of the solution of a matrix Riemann-Hilbert problem, and applying a steepest-descent approach. This approach originates in the derivation of the long-time asymptotic behavior of the solution of integrable nonlinear PDEs, and more recently has provided ground-breaking results in the analysis of the semi-classical limit of systems such as KdV and NLS, as well as in approximation theory and random matrix theory.

This is joint work with Ken McLaughlin and Xin Zhou.