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## Orthogonal Rational Functions

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### Abstract

Orthogonal Rational Functions (ORFs) are meromorphic functions with no poles in the extended complex plane outside of the real pole set  $\{\alpha_1, \dots, \alpha_K\}$ , with  $0 \leq |\alpha_k| < +\infty$ ,  $k = 1, \dots, K$ , which satisfy a system of orthogonality conditions; in particular, ORFs orthogonal with respect to varying exponential weights  $\omega(z) = \exp(-\mathcal{N}V(z))$ ,  $\mathcal{N} \in \mathbb{N}$ , where  $V: \mathbb{R} \setminus \{\alpha_1, \dots, \alpha_K\} \rightarrow \mathbb{R}$  is real analytic and satisfies certain 'growth conditions' at  $\infty$  and at  $\alpha_k$ ,  $k = 1, \dots, K$ , are considered. The principal question addressed in this talk is: how does one obtain asymptotics in a *double-scaling limit* as  $\mathcal{N}$  and  $n$  (= the 'degree' of the ORF) tend to infinity in such a way that  $\mathcal{N}/n = 1 + o(1)$  for  $z$  anywhere in the extended complex plane and  $k = 1, \dots, K$ ? This will be addressed by considering the FOKAS-ITS-KITAEV reformulation of the ORF problem as a matrix Riemann-Hilbert problem on  $\mathbb{R}$ , and then extracting the asymptotic behaviour via the DEIFT-ZHOU non-linear steepest-descent method. Possible applications of ORFs to Multi-Point Padé Approximants, Random Matrix Theory, and Painlevé Transcendents will be discussed.

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DATE: Friday, April 13, 2007  
TIME: 3:00 - 4:00pm  
PLACE: Math and Physics Building, Room 204