The asymptotic location of the zeros of a sequence of hypergeometric functions

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In recent papers, Kuijlaars and Martínez-Finkelshtein and their co-authors have used Riemann-Hilbert methods to derive the asymptotic zero distribution of Jacobi polynomials $P_n^{(\alpha_n,\beta_n)}$ when the limits

$$A = \lim_{n\to\infty} \frac{\alpha_n}{n} \text{ and } B = \lim_{n\to\infty} \frac{\beta_n}{n}$$

exist and lie in the interior of certain specified regions in the AB-plane. We prove that the zeros of ${}_2F_1\left(-n, \frac{n+1}{2}; \frac{n+3}{2}; z\right)$ asymptotically approach the section of the lemniscate $\left\{z : |z(1-z)^2| = \frac{4}{27}; \mathrm{Re}(z) > \frac{1}{3}\right\}$ as $n \to \infty$. Our result corresponds to one of the transitional or boundary cases for Jacobi polynomials in the Kuijlaars Martínez-Finkelshtein classification.