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Riemann–Hilbert problems: Malgrange, Schlesinger and Sato

Marco Bertola, Concordia University, Montreal

Abstract

Given any (sufficiently well-behaved) family of Remann–Hilbert problems (RHP) where the jump matrices depend arbitrarily on deformation parameters, we can construct a one-form Ω on the deformation space (Malgrange's differential).

Such a one-form has a pole where the deformation family meets the Malgrange Theta divisor, namely, the set of unsolvable RHP.

The differential Ω fails to be closed in general, but when it does the formula $\tau := e^{\int \Omega}$ defines locally a function that vanishes precisely on Θ.

We then introduce the notion of Schlesinger discrete transformation: it means that we allow the solution of the RHP to have poles (or zeros) at prescribed point(s). Interestingly, even if Ω is not closed, the difference of Ω evaluated along solution of the original RHP and the Schlesinger transformed RHP is closed off Θ; in fact, such difference is shown to be the logarithmic differential (on the deformation space) of a function. This yields a Sato-like formula for the solution of the RHP.

For certain specific RHPs the tau function coincides with the one introduced in the eighties by Jimbo, Miwa and Ueno; in the present formulation we can write variational formulae w.r.t. the (extended) monodromy data as well.

Some applications include (some of them are known by other means): variational equations for Painleve' transcendents w.r.t. Stokes' data, variational equations for finite Hankel/Toeplitz determinants for discontinuous symbols.

Date: Thursday, March 19, 2009

Time: 11:30 AM

Place: MAP 318

Everyone is cordially requested to attend.